A structural Bayesian VAR for model-based fan charts

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Abstract

Inflation forecast uncertainty is of importance for a wide range of agents in the economy, central banks in particular. Ways to describe and account for this uncertainty in a consistent manner have received increasing attention of late, in part due to the growing number of inflation-targeting central banks. This paper develops a large structural VAR for the Swedish economy and estimates it in a Bayesian framework. The methodology permits not only structural interpretation and analysis but offers a natural way to formalise forecast uncertainty, as the posterior predictive density from the model has the interpretation of a fan chart.

JEL Classification: C32, C53, E47, E52

Keywords: Inflation, Forecasts, Uncertainty

[#] The views expressed in this paper are solely the responsibility of the author and should not be interpreted as reflecting the views of the executive board of Sveriges Riksbank.

1. Introduction

Inflation forecasts, and the uncertainty surrounding them, are of interest to a large number of economic agents, not least because unexpected inflation redistributes wealth between borrowers and lenders and affects profits when prices are not completely flexible. Estimates of inflation forecast uncertainty which receive most attention in economies for which they are published are so-called inflation *fan charts* produced by inflation-targeting central banks, such as the Bank of England and Sveriges Riksbank.

The present methods used to generate such inflation fan charts have, however, been criticised recently by a number of authors, among them Leeper (2003) and Clements (2004), and the criticisms are several. The Bank of England's inflation fan charts express the subjective probability distribution for inflation held by the Monetary Policy Committee. However, there appear to be several layers of informal judgement included in the analysis and the procedure by which the analysis is conducted is unclear, leading some to question the statistical foundation of this method. Similarly, Sveriges Riksbank's fan charts are built on a shaky foundation whose main shortcoming is that no mechanism exists to guarantee internal consistency. For example, the method exhibits the peculiar feature that both inflation and the factors determining it *can always be more (or less) uncertain than usual*. Given these shortcomings, there is value in aiming to develop fan chart methods that are less *ad hoc* and better grounded in statistical theory.

Cogley *et al*. (2005) made a substantive contribution to improving fan chart methodology by employing a reduced-form Bayesian VAR with parameter drift to generate fan charts for U.K. inflation. This paper extends previous work by generating fan charts for a number of macroeconomic variables from a structural VAR estimated with Bayesian methods. The fan charts, which are given by the posterior predictive density from the model, are thereby generated in a model-consistent manner with a solid foundation in statistical theory. Compared to Cogley *et al*. (2005), we take the analysis forward in two directions; first and foremost, the model used to conduct the analysis in this paper is a structural VAR. This allows structural interpretation of the shocks in the system, extending the use of the model from forecasting and fan charts to policy analysis. Second, the model is set in a small open economy, thereby addressing issues that a country such as Sweden faces. Overall, the suggested framework means that forecasting and policy analysis are brought closer together and fan charts integrated into the framework. As such, this paper is related to other parts of the central banking literature in which fan charts are addressed in a model-consistent way; see for example Svensson and Williams (2005).

The paper is organised as follows: Section two presents a structural VAR for the Swedish economy. Section three outlines the estimation of the model and the properties of the estimated model, such as impulse response functions and out-of-sample point forecasting ability. Section four presents the fan charts generated by the model, relates the inflation fan chart to the present method used by Sveriges Riksbank and addresses potential shortcomings. Finally, Section five concludes.

2. A structural VAR for the Swedish economy

Structural VAR models have become increasingly popular over the last two decades and are nowadays standard tools for economic analysis. Whilst these models can be specified and estimated in many different ways, the common feature of these models is that a number of identifying assumptions are made in order to be able to make statements regarding a number of structural shocks in the economy. The issue of identification has been dealt with in different ways in the literature; for a range of alternative approaches, see for example Sims (1980), Bernanke (1986), Shapiro and Watson (1988), Blanchard and Quah (1989), Crowder and Wohar (2004), Groen (2004) and Uhlig (2005).

The structural model that will be used in this paper belongs to the family of *K*-models, using the terminology of Amisano and Giannini (1997). This means that restrictions are put on the coefficients of contemporaneous variables in particular structural equations.[1] Turning to the specification of the model that we aim to estimate in more detail, it is given by

$$\mathbf{G}(L)\mathbf{x}_t = \boldsymbol{\mu} + \boldsymbol{\Theta}\mathbf{D}_t + \boldsymbol{\varepsilon}_t, \qquad (1)$$

where $\mathbf{G}(L)$ is a lag polynomial of order p, $\mathbf{x}_t$ is an nx1 vector of macro variables, $\mathbf{D}_t$ is a

[1] Studies using *K*-models include Bernanke (1986), Kim and Roubini (2000), Camarero *et al*. (2002) and Villani and Warne (2003).

kx1 vector of dummy variables $\boldsymbol{\varepsilon}_t$ an nx1 vector of independent multivariate normal distributed error terms fulfilling $E(\boldsymbol{\varepsilon}_t) = \mathbf{0}$ and $E(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t') = \mathbf{I}_n$. However, the structural model cannot be directly estimated. In order to recover the structural model, we must first estimate a reduced form model given by

$$\mathbf{A}(L)\mathbf{x}_t = \boldsymbol{\delta} + \boldsymbol{\Phi}\mathbf{D}_t + \mathbf{v}_t \qquad (2)$$

where $\mathbf{A}(L)$ is a lag polynomial of order p with $\mathbf{A}_0 = \mathbf{I}_n$ and $\mathbf{v}_t$ an nx1 vector of independent multivariate normal distributed error terms fulfilling $E(\mathbf{v}_t) = \mathbf{0}$ and $E(\mathbf{v}_t \mathbf{v}_t') = \boldsymbol{\Sigma}$. The relation between structural and reduced form is then used to recover the structural model. The most fundamental part when doing this is the relation between the structural and reduced form residuals. Since the models imply that $\mathbf{v}_t = \mathbf{G}_0^{-1}\boldsymbol{\varepsilon}_t$, we use the expression

$$\boldsymbol{\Sigma} = \mathbf{G}_0^{-1}\left(\mathbf{G}_0^{-1}\right)' \qquad (3)$$

to recover $\mathbf{G}_0$. In order to achieve identification, we must impose at least $n(n-1)/2$ restrictions on the system; these restrictions are given in the $\mathbf{G}_0$ matrix and will be discussed in more detail below.

In order to further present the structural model, we first need to introduce the variables included in the system. Nine endogenous variables are modelled in the VAR. We define

$$\mathbf{x}_t = \left(c\Delta y_t^f \quad \pi_t^{cpi,f} \quad i_t^f \quad u_t \quad c\Delta y_t \quad c\Delta w_t \quad \pi_t^{cpi} \quad i_t \quad cq_t\right)'$$

where $c = 100$, y_t^f is the logarithm of GDP in fixed prices for the foreign economy, $\pi_t^{cpi,f}$ is twelve month ended CPI inflation for the foreign economy and i_t^f is the three month treasury bill rate for the foreign economy.[2] Turning to the Swedish variables, u_t is the open

[2] Foreign GDP and CPI have been trade weighted according to the TCW index. The foreign interest rate has been weighted using a subset of the countries included in the TCW index due to missing data for some countries.

unemployment rate, y_t is the logarithm of GDP in fixed prices, w_t is the logarithm of wages, π_t^{cpi} is twelve month ended CPI inflation and i_t is the three month treasury bill rate. Finally, q_t is the logarithm of the trade weighted real exchange rate, given as SEK per foreign currency. All variables except interest rates and the real exchange rate have been seasonally adjusted.

As we want the variables in the system to be stationary, we have taken the first difference of wages and foreign and Swedish GDP. The stationarity of the variables has been investigated using the Augmented Dickey-Fuller test (Said and Dickey, 1984) and the results are presented in Table A1 in Appendix A. Clearly, not all of the variables which have been modelled in levels in the system are judged stationary by the unit root tests. We nevertheless choose to model these variables in levels based on theoretical arguments and previous empirical findings; see for example Österholm (2004).[3]

We have now presented the model's general form and the variables included. It should be noted that compared to previous work in this field, the model employed in this paper introduces both structure and a high dimension in the system. However, this adds a layer of complication relative to the analysis in for example Cogley *et al*. (2005) where a three variable reduced form VAR was used. We therefore have to trade off something else in order to make estimation possible. As can be seen from equation (1), we choose to estimate the model using constant parameters over time. Given the problems of identifying drifting parameters in general, we consider it reasonable to leave this outside the model. In particular, since the number of parameters grows extremely fast with the dimension of the system we believe that this is a relevant trade off, despite the fact that parameter drift employed by Cogley *et al*. (2005) is an appealing feature of a model.[4]

Turning to the identification of the model, it is most easily presented by the relation $\boldsymbol{\varepsilon}_t = \mathbf{G}_0 \mathbf{v}_t$ and this is given in equation (4). As pointed out above, we need at least

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[3] We are also well aware of the low power of Dickey-Fuller type tests when the investigated series have roots close to – but less than – unity or when they are subject to structural breaks; see for example Froot and Rogoff (1995) and Perron (1989).

[4] Not allowing for time-varying parameters in the model implies – apart from the fact that the model could be mis-specified – that one potential source of uncertainty has been left out. This could clearly affect the fan charts from the model. As pointed out by Cogley *et al*. (2005) though, constant parameters are a reasonable assumption from a forecasting point of view as long as the forecasting horizon is short – the reason for this being that parameter drift typically is very small. Support for usage of constant parameters can also be found in Doan *et al*. (1984) who argue that the improvement in forecast performance from using time-varying parameters typically is very small.

$n(n-1)/2$ restrictions to identify the system. However, as shown by Lucchetti (2006), this is not sufficient for identification; in order to assure identification, a structure condition must also be met. That the present model actually is identified has been investigated using the algorithm suggested by Lucchetti (2006).

$$\begin{bmatrix} \varepsilon_{d^f} \\ \varepsilon_{c^f} \\ \varepsilon_{ir^f} \\ \varepsilon_{\theta} \\ \varepsilon_{d} \\ \varepsilon_{ws} \\ \varepsilon_{ps} \\ \varepsilon_{ir} \\ \varepsilon_{q} \end{bmatrix} = \begin{bmatrix} g_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & g_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & g_{23} & g_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & g_{44} & g_{45} & 0 & 0 & 0 & 0 \\ g_{51} & g_{52} & g_{53} & 0 & g_{55} & 0 & g_{57} & g_{58} & 0 \\ g_{61} & g_{62} & 0 & g_{64} & g_{65} & g_{66} & g_{67} & 0 & 0 \\ g_{71} & g_{72} & g_{73} & 0 & g_{75} & g_{76} & g_{77} & 0 & 0 \\ 0 & g_{82} & g_{83} & g_{84} & 0 & g_{86} & g_{87} & g_{88} & g_{89} \\ g_{91} & g_{92} & g_{93} & g_{94} & g_{95} & g_{96} & g_{97} & g_{98} & g_{99} \end{bmatrix} \begin{bmatrix} \nu_{\Delta y^f} \\ \nu_{\pi^{cpi,f}} \\ \nu_{i^f} \\ \nu_{u} \\ \nu_{\Delta y} \\ \nu_{\Delta w} \\ \nu_{\pi^{cpi}} \\ \nu_{i} \\ \nu_{q} \end{bmatrix} \qquad (4)$$

The chosen system could be described as an open economy model inspired by the model used by Blanchard (1989) in a study on U.S. data. To Blanchard's model have three equations for the foreign economy and a real exchange rate equation been added. In specific, equations one to three give – in a highly reduced form – aggregate demand, Phillips curve and monetary policy reaction function for the foreign economy. The fifth equation in the system represents Swedish aggregate demand whereas equations four, six and seven describe the supply side of the Swedish economy in greater detail; these represent Okun's law, a wage setting equation and a price setting equation respectively. Equation eight represents a monetary policy reaction function and, finally, equation nine an "arbitrage condition" for the real exchange rate.[5]

In accordance with the above presented system, nine structural shocks are identified. Three of these are for the foreign economy, namely a foreign aggregate demand shock (ε_{d^f}), a foreign cost-push shock (ε_{c^f}) and a foreign monetary policy shock (ε_{ir^f}). For the Swedish economy, we identify an aggregate supply shock (ε_{θ}), an aggregate demand shock (ε_{d}), a wage setting shock (ε_{ws}), a price setting shock (ε_{ps}), a monetary policy shock and, finally, a

[5] Note that both the foreign central bank and the Riksbank are assumed unable to react to innovations in GDP contemporaneously. This is justified by the substantial publication lag in GDP numbers; see for example Leeper *et al*. (1996).

real exchange rate shock (ε_q).[6]

3. Estimation and results

Quarterly data on the variables ranging from 1980Q2 to 2004Q4 were supplied by Sveriges Riksbank. Apart from the variables in $\mathbf{x}_t$ – which were defined above – we let $\mathbf{D}_t$ be a dummy variable which takes on the value 1 between 1980Q1 and 1992Q4 and 0 otherwise.[7] Lag length in the model was set to $p = 4$.

The model is estimated using Bayesian methods and priors on dynamics in the model follow the standard modelling approach, as they take their starting point in a Minnesota prior.[8] A minor modification is, however, introduced regarding the dynamics as block exogeneity of the foreign economy not is enforced in the model with probability one; instead, the exogeneity restriction is controlled using an additional hyperparameter.[9] Priors on the constant term, the dummy variable and $\mathbf{G}_0$ all follow those in Villani and Warne (2003) with diffuse normal priors.[10]

The numerical evaluation of the posterior distributions is conducted using the Gibbs sampler; see for example Tierny (1994). Convergence of the chain appears to be fast and reliable, an issue which has been confirmed using CUSUM plots.[11] Given this, the burn-in sample is set to only 2 000 draws and the analysis is then performed on the 20 000 draws following these. As is well-known, the chain is serially dependent but there has been no thinning of it. Whilst

[6] Most of the shocks are standard and straight forward to interpret. Regarding the shocks that could be less obvious to interpretate, the aggregate supply shock corresponds to changes in productivity and/or labour supply, the wage setting shock could for instance reflect changes in bargaining power on the labour market and the price setting shock can correspond to higher input prices or changes in mark-ups.

[7] This might seem like an overly simplistic way to model the regime shift in Swedish monetary policy which occurred in the early 1990s. However, it appears to have worked well when used previously in the literature; see for example Jacobson *et al*. (2001) and Villani and Warne (2003). Moreover, since several of the countries that have large weights in the TCW index also experienced policy changes around the same period – for example Germany, Norway and the U.K. – we also let the dummy variable affect the foreign economy. Changing this assumption, thereby letting the dummy affect the Swedish economy alone, has negligible effects on the results.

[8] See for example Litterman (1986) and Robertson and Tallman (1999).

[9] See for example Villani and Warne (2003).

[10] As the model is invariant to sign switches in the equations in $\mathbf{G}_0$ – see for example Sims and Zha (1999) – we must also employ some kind of normalisation to the system. In this paper we employ the Waggoner and Zha (2003) normalisation which has been shown to have good properties both theoretically and empirically.

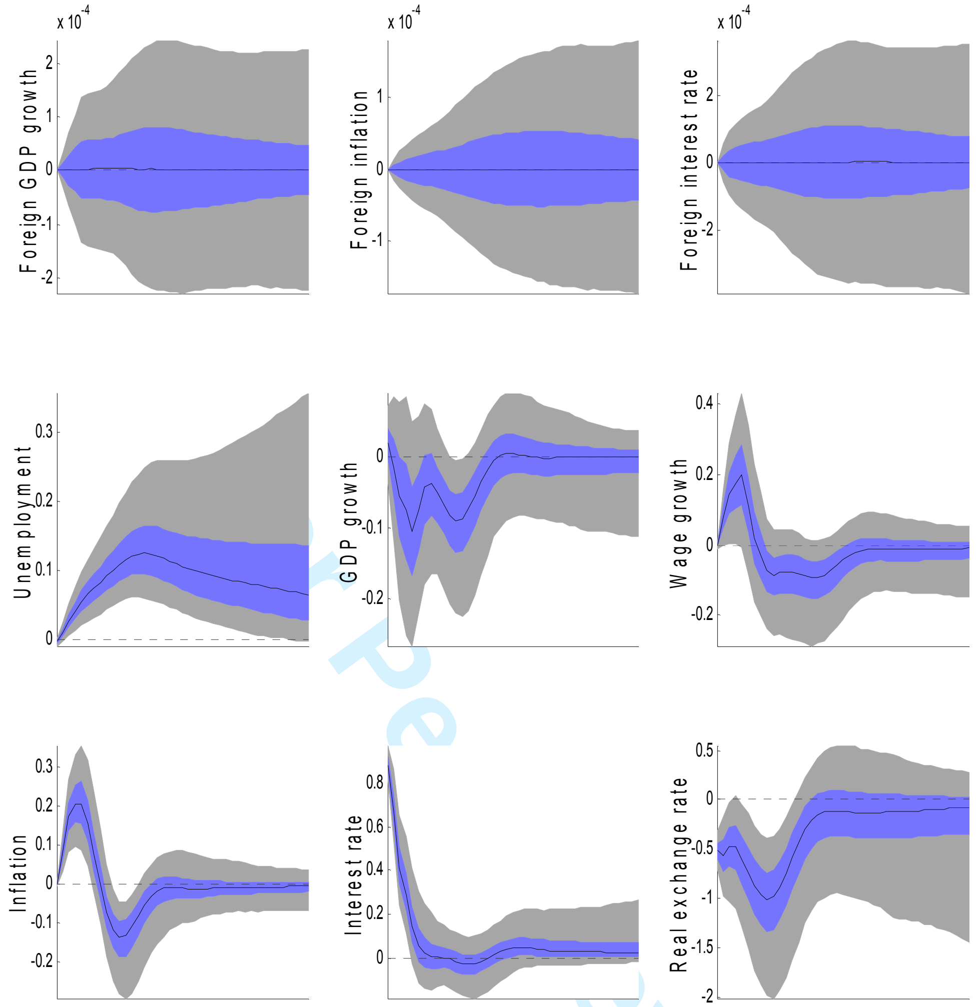
[11] Results not reported but available upon request.

this could be done in order to increase efficiency, it is largely a matter of taste since at convergence the draws are identically distributed according to the posterior distribution; see for example Gelman *et al*. (2004). Based on this sampling we can conduct simulation-based inference for a range of functions from the model. This will next be the focus of our attention.

3.1 Impulse response functions

Whilst the purpose of the model primarily is to generate a model-based fan chart for inflation, it is initially of interest to establish whether the properties of the model are in line with our expectations based on economic theory. One important feature of this analysis is to investigate the properties of the impulse response functions from the model. In a model where monetary policy analysis is the primary objective, the effects of a domestic monetary policy shock are obviously of particular interest. This section will therefore focus on responses to this shock. Figure 1 presents the impulse response functions of a Swedish monetary policy shock to the variables of the system. Initially, we can note that the foreign economy is block exogenous in practice in line with our expectations; all effects of the Swedish shocks on the foreign economy are zero. Regarding the effects on Swedish variables, we can see that unemployment rises and GDP growth falls as one would expect. The significance of the latter effect may be discussed though, as could the effect on wage growth which is a touch anomalous with the point estimate indicating an increase initially. We can note that the cumulative effect of the median impulse response is correct though. This gives us the sum as the horizon goes to infinity and provides the effect of a permanent change to the interest rate.

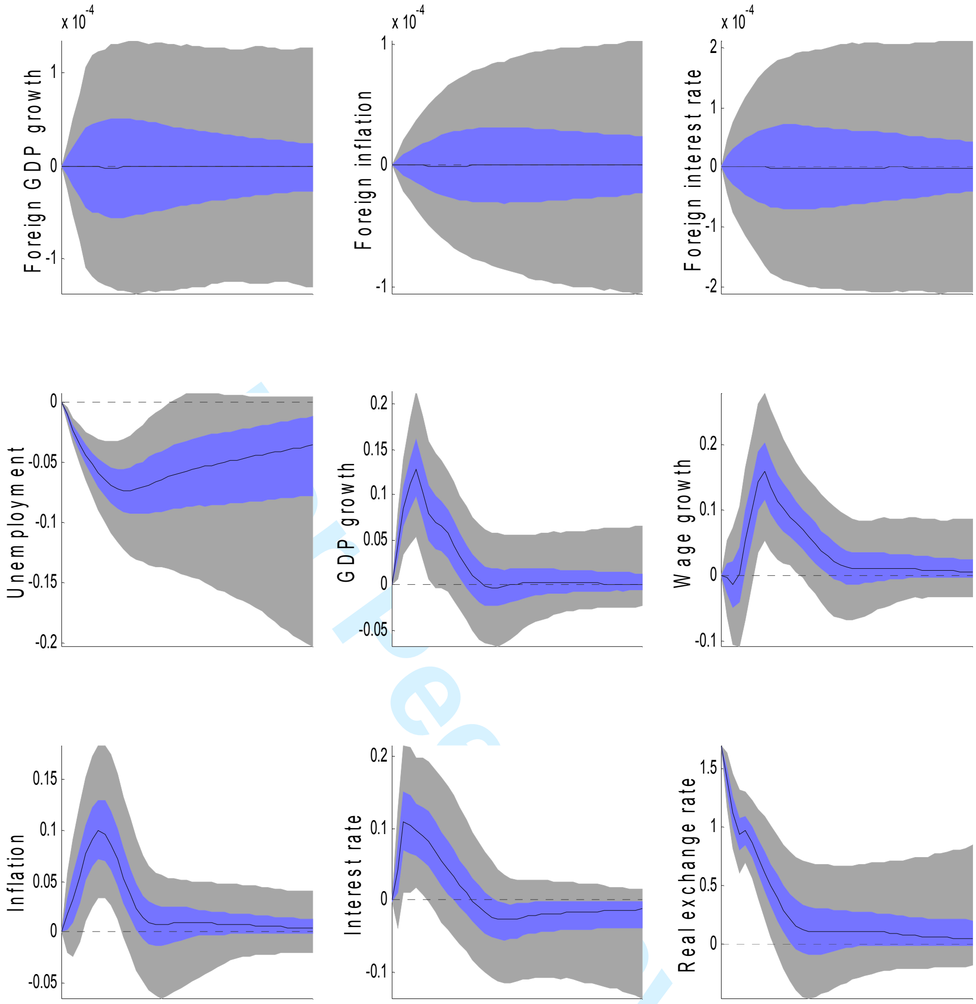
Figure 1. Impulse response functions of variables in the system with respect to Swedish monetary policy shock.



Black line is the median impulse. Coloured bands are 50 and 90 percent confidence bands.

Turning to inflation we find what appears to be a traditional price puzzle – see for example Sims (1992) and Creel (2006) – since inflation rises significantly before it falls. However, this is partly due to the fact that CPI is being used as the price measure. The way the CPI is constructed, there is a mechanical explanation to a part of this effect as housing expenses directly depend on the interest rates. This is confirmed by using exactly the same model except for the price measure. If we instead turn to the UND1X index – which excludes this mechanical effect as the interest rate component has been removed from CPI – there is no significant price puzzle in the model anymore. The impulse response functions for the alternative model are shown in Figure A1 in Appendix C.

Figure 2. Impulse response functions of variables in the system with respect to Swedish real exchange rate shock.



Black line is the median impulse. Coloured bands are 50 and 90 percent confidence bands.

Apart from the price puzzle, many structural VAR models tend to suffer from what is known as an exchange rate puzzle; see for example Grilli and Roubini (1996). The present model shows no sign of such a puzzle as the exchange rate significantly appreciates on impact when a monetary policy shock hits the system.

To further investigate the open economy aspects of the model, we also look at the response of a real exchange rate shock to the system. This is given in Figure 2 and as we can see the effects are all in line with our expectations. As before, there is no effect on the foreign economy but for the domestic variables we can see that unemployment decreases, whereas

GDP growth, wage growth, CPI inflation and the interest rate all rise. Clearly, all of these effects are in line with a traditional exchange rate channel. Summing up the effects of monetary policy and real exchange rate shocks, we find that the model generates impulse responses that largely are in line with expectations. As the model seems reasonably well-behaved, we think that the model seems fit for policy analysis.

3.2 Point forecasts

Given that we think that the model has sensible properties in terms of its impulse response functions, our next step is to evaluate its forecasting ability. A reasonable performance with respect to the out-of-sample point forecast accuracy seems like a relevant requirement for the model to be taken seriously. The model's forecasts at horizons one, four and eight quarters for the key variables y_t^a, π_t^{cpi} and i_t are evaluated using rolling out-of-sample forecasts.[12] The out-of-sample forecasts are generated beginning in 2000Q1; the last forecasts being evaluated are based on the model estimated on data including 2004Q3. This implies that there will be seven (three) more forecasts to evaluate at the one-quarter horizon than at the eight-quarter (four-quarter) horizon.

Table 1. RMSEs from out-of-sample forecast exercise.

Variable	Horizon in quarters	VAR	Naïve	Riksbank	VAR – mean adjusted
π_t^{cpi}	1	0.549	0.549	0.501	0.518
	4	1.221	1.246	0.939	0.690
	8	1.165	1.503	1.241	1.454
y_t^a	1	0.678	0.379	-	0.493
	4	2.804	1.972	-	1.810
	8	2.356	1.669	-	1.484
i_t	1	0.519	0.264	-	0.379
	4	1.604	0.746	-	1.972
	8	2.612	1.294	-	1.669

[12] y_t^a is the growth rate in Swedish GDP over the last four quarters. As the VAR is specified in first and not fourth differences for Swedish GDP, the forecasts are therefore first summed up to fourth differences and then evaluated.

The criterion used for evaluation is the root mean square error (RMSE) and results from this exercise are presented in Table 1. RMSEs for the VAR are given in column three of the table and RMSEs for a naïve forecast – which is optimal if the process being forecasted is a univariate random walk – are given in column four. For further comparison are the RMSEs based on Sveriges Riksbank's official forecasts in the *Inflation Report* shown in column five.

Strictly speaking, the RMSEs from Sveriges Riksbank's forecasts are not completely comparable to the other forecasts for several reasons. First, the inflation forecasts from the VAR are evaluated against actual inflation numbers based a seasonally adjusted CPI series. Sveriges Riksbank's inflation forecasts on the other hand are evaluated against actual inflation numbers based on the original CPI series. Second, it should be noted that despite the fact that equally many forecast errors are recorded at all horizons for all models, there is a small difference in the information set used. In particular, even though the difference generally is small, the timing of the forecasts in the *Inflation Reports* of Sveriges Riksbank does not perfectly correspond to the timing of the VAR and naïve forecasts.[13] Finally, Sveriges Riksbank did under the period analysed rely the assumption that the repo rate would remain constant during the forecast horizon. Such an assumption could clearly lead to suboptimal forecasts and would make us argue that better forecasts should have been generated without this assumption. However, whether this assumption has been properly used in practice has recently been questioned by for example Faust and Henderson (2004) and Faust and Leeper (2005). There is evidence that the assumption of a constant repo rate has not been used consistently and the forecasts of Sveriges Riksbank might therefore have less of a disadvantage in practice than in theory. However, despite the potential advantages and disadvantages that the forecasts of Sveriges Riksbank may have had relative to those from the VAR, we think that they still can provide at least a reference point for a comparison.

Looking at the results in Table 1, we find that they are not overly flattering for the VAR when evaluating the forecasts performance for y_t^a and i_t. The VAR is actually outperformed by the naïve forecast at all horizons. For π_t^{cpi} though, it appears that the VAR is doing reasonably well as its RMSEs are smaller than, or equal to, those of the naïve forecast.

[13] The fact that Sveriges Riksbank also has to use real-time data – whereas the VAR relies on *ex post* data in the out-of-sample forecast exercise – is a related issue. For a discussion regarding issues on real-time data, see for example Croshoure and Stark (2002) and Orphanides and van Norden (2002).

Comparing the forecasts from the VAR to those of Sveriges Riksbank, it can be seen that the VAR outperforms Sveriges Riksbank at the eight quarter horizon but not at the two shorter horizons. A potential explanation for the good performance of Sveriges Riksbank's forecasts at the shorter horizons could be the use of judgement – a feature that the VAR lacks. Numerous forecasters argue that judgement should be included in the forecasting process as it typically is assumed to improve forecasts, in particular at the short horizons.[14] The differences are not large at any horizon though and should therefore not be over-interpreted. Rather, it seems promising that the VAR's inflation forecasts are approximately as good those of Sveriges Riksbank.

Whilst the out-of-sample forecast exercise here is reasonably short, the results regarding GDP growth and the interest rate indicate that there might be some minor problems with the VAR's forecasts. One potential problem which is common for both VARs and univariate processes is that they occasionally tend to converge to the "wrong" level. The forecasts will converge to the estimated unconditional means of the process and these are a function of the estimated parameters in $\boldsymbol{\mu}$, $\mathbf{G}(L)$ and $\boldsymbol{\Theta}$. If the estimates of these parameters are "unfavourable", the estimated unconditional means can be far from what we expect it to be. For instance, we would expect the forecast of π_t^{cpi} to level out at two percent since this is the target of Sveriges Riksbank.

One way to incorporate information about the unconditional means of the variables in the system is to use the framework in Villani (2005). This recently developed method relies on estimation of a Bayesian VAR in mean-adjusted form and we accordingly specify the model as

$$\mathbf{G}(L)(\mathbf{x}_t - \boldsymbol{\alpha} - \boldsymbol{\Xi}\mathbf{D}_t) = \boldsymbol{\varepsilon}_t \qquad (5)$$

which allows us to put informative normal priors on the unconditional means; see Villani (2005) for details.[15] Working in this mean-adjusted form and adding informative priors on the unconditional means – but keeping the model unchanged otherwise, including

[14] For a discussion on this issue, see for example Lawrence *et al*. (1986) and McNees (1990) and Svensson (2005).

[15] For the model in equation (5) the unconditional means for the periods 1980Q2 to 1992Q4 and 1993Q1 to 2004Q4 are given by $\boldsymbol{\alpha} + \boldsymbol{\Xi}$ and $\boldsymbol{\alpha}$ respectively.

identification – we get the RMSEs in column six of Table 1. It can be seen that the VAR's forecasting ability generally improves and it tends to outperform the naïve forecast when it comes to π_t^{cpi} and y_t^a. Regarding π_t^{cpi}, we also see that the mean-adjusted VAR largely is on par with Sveriges Riksbank's forecasts. However, for i_t it is still the case that the naïve forecast is better than that of the VAR. Given the slow mean reversion of the variables considered though, we know that the naïve forecast tends to be difficult to beat and therefore think that the above results favour the viewpoint that the model's forecasting ability is quite acceptable.[16]

4. The inflation fan chart

Typically, the focus when it comes to forecasting has been on the point estimate from a particular model. Whilst we think that this is an important feature of any model that want to be taken seriously, this paper is mainly concerned with describing forecast uncertainty and thereby the distribution of forecasts.

The distribution of forecasts from the VAR model is generated in a simple way. For every draw from the posterior distribution, a sequence of shocks are drawn and used to generate future data. Based on this routine, we get as many paths for each variable as we have iterations in the Gibbs sampling algorithm. For each variable in the system, we can plot chosen percentiles from the marginalised distributions. We thereby get model generated fan charts which take both shock and parameter uncertainty into account in a straight forward way. Clearly, numerical analysis is not necessary in many models to achieve this. However, it offers a convenient approach since it basically always can be conducted.

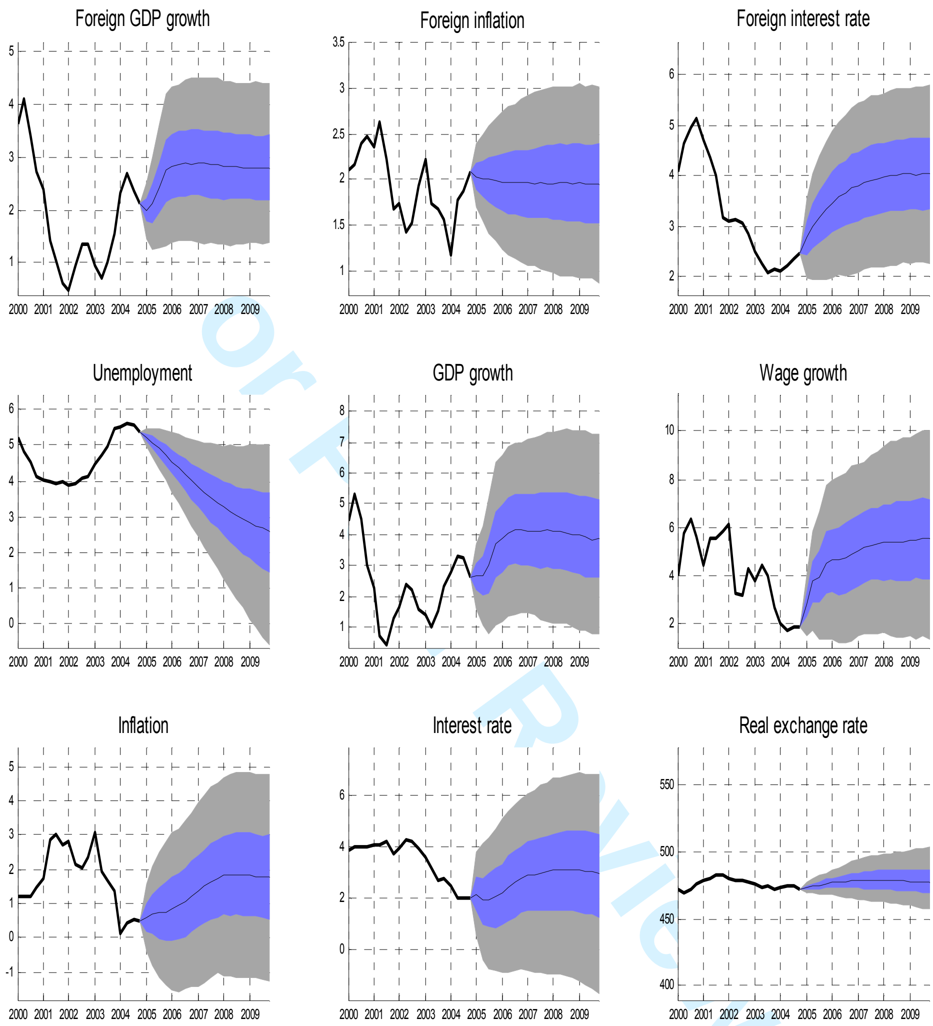
The posterior predictive density gives us fan charts – not only for inflation, even though this is our main interest in this paper – but for all variables in the model. Figure 3 presents the fan charts generated by the model under investigation in this paper.[17] The median forecast (black line) is given together with 50 and 90 prediction intervals in order to be able to compare the inflation forecasts from the model with those presented in Sveriges Riksbank's *Inflation*

[16] Persistence estimates for the different time series are given in Table A2 in Appendix B. Note though that y_t^a is more persistent than Δy_t due to the overlapping nature of the former series.

[17] Note that for variables specified in first differences, the numbers in Figure 3 all refer to growth rates over the last four quarters. As was the case for Swedish GDP in the out-of-sample forecast evaluation above, the forecasts are therefore summed up to fourth differences.

Report. As can be seen from the figure, the model predicts that inflation will rise slowly during the forecast horizon and – judging by the median forecast – approach the target level of two percent by early 2008.

Figure 3. Fan charts for variables in the system.

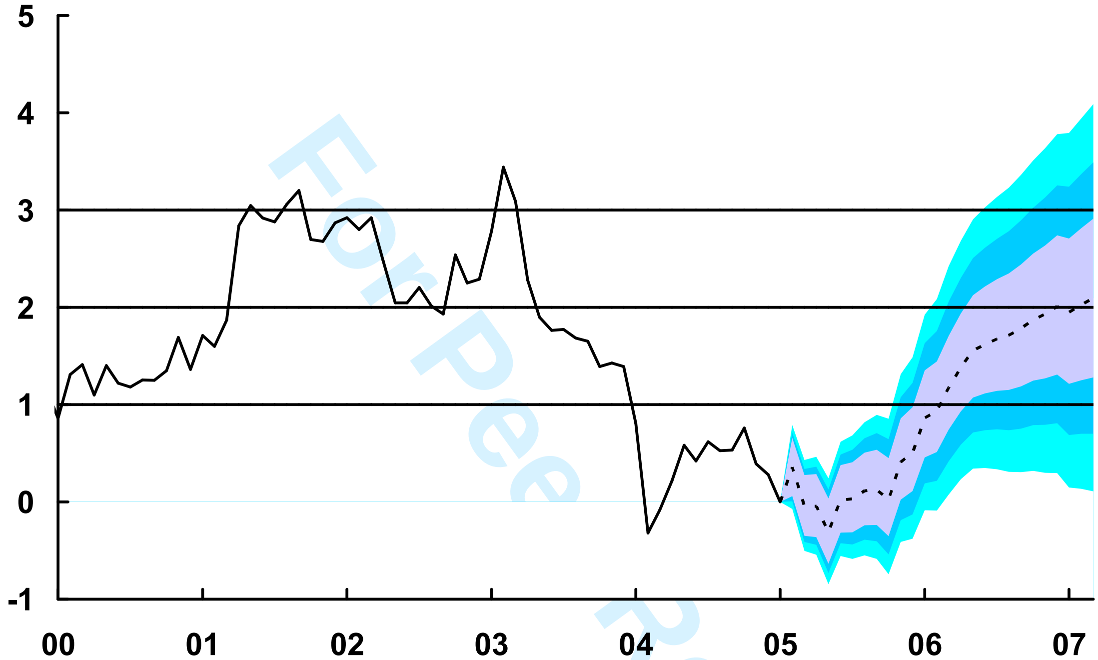


Black line is the median forecast. Coloured bands are 50 and 90 percent confidence bands.

Turning to the issue of forecast uncertainty though, we find that the inflation forecast is associated with a considerable amount of this. As a reference between the present model – which builds on a weighting of split-normal distributions (Blix and Sellin, 1998) – and the structural Bayesian VAR used in this paper, Figure 4 presents the inflation fan chart generated by Sveriges Riksbank in its *Inflation Report 2005:1*. It is obvious from Figures 3 and 4 that the VAR generates a wider inflation fan chart than the present method used by

Sveriges Riksbank. It should be noted though that, unlike the VAR used in this paper, the method used by Sveriges Riksbank allows for time-varying forecast uncertainty. To ensure that the fan chart presented in Figure 4 was not a particularly narrow (or wide) fan chart, we also investigated the width of the inflation fan charts in the other *Inflation Reports*. This showed us that the fan chart in Figure 4 is reasonably representative.

Figure 4. Sveriges Riksbank's fan chart for inflation in Inflation Report 2005:1.



Dashed black line is the mode forecast. Coloured bands are 50, 75 and 90 percent confidence bands.

But does the method presently used by Sveriges Riksbank underestimate inflation forecast uncertainty or does the VAR overestimate it? The most likely answer to this question is that the truth lies somewhere in between. Critique against the present method has been put forward in for example Leeper (2003) where it is indicated that there might be systematic problems with it. One problem is that in the weighting of the split-normal distributions, the weights are taken as given. This implies that parameter uncertainty largely is ignored in the model, which clearly is a shortcoming. The consequence of this omitted parameter uncertainty is an underestimation of the forecast variance. However, it is highly unlikely that this fact alone can explain the discrepancy between the two models. Another potential reason why the method presently used by Sveriges Riksbank generates a narrower inflation fan chart is simply that the shock variance to inflation and/or its determinants could be underestimated. A potential problem with the VAR on the other hand is that shock uncertainty might be overestimated. If the high level of inflation in the 1980s also was associated with a large variance, averaging over these regimes could clearly generate too wide prediction intervals. To settle this issue an evaluation in which one evaluates the forecast densities is needed. This is, however, difficult to perform in practice since the number observations for such an analysis would be too small to be meaningful and therefore beyond the scope of this paper.[18]

To sum up the discussion regarding the fan charts, it should be clear that even though the framework suggested in this paper has potential shortcomings – which could be due to the choice of using constant parameters over time – it still has several advantages over the methodologies presently used by the Bank of England and Sveriges Riksbank. First, we would like to argue that accounting for both shock and parameter uncertainty is an appealing feature of the method. Second, the usage of a structural VAR gives a general equilibrium perspective to the analysis. This should be compared to the present method employed by Sveriges Riksbank which is fundamentally partial in its analysis and thereby highly likely to be plagued by inconsistencies. Finally, we would like to point out the benefits of the joint predictive density for all the variables in the system generated in this framework. Faced with the joint predictive density it is easy to study joint probability questions of the type investigated by Leeper and Zha (2003). Knowing for example what the probability of inflation above target *and* negative output growth at a particular horizon is should clearly be of interest to policy makers. This kind of analysis is excruciatingly difficult – or even impossible – in the frameworks presently used at the Bank of England or Sveriges Riksbank. The fact that it is readily conducted in the framework suggested in this paper must therefore be seen as another major benefit of the method.

5. Conclusions

This paper has presented a structural Bayesian VAR for the Swedish economy with several appealing features. Fan charts can be generated in a straightforward manner for variables in the system, with a particular focus here upon inflation. The proposed framework serves as a formal and model-consistent methodology in contrast to methods presently used to generate fan charts at the Bank of England and Sveriges Riksbank. Another appealing feature is that

[18] See for example Diebold *et al*. (1998) and Diebold *et al*. (1999) for a discussion on how to evaluate density forecasts.

the model can be employed for both forecasting and policy analysis. Whilst the latter has not been the focus of attention in this paper, it represents an advantage over analyses conducted with reduced-form VARs, such as that of Cogley *et al*. (2005).

The analysis in this paper has all been carried out under the assumption that we rely solely upon the model's unconditional forecast. The framework, however, can be extended to allow variables to be conditioned upon, for instance in order to incorporate external information, take into account early data releases or evaluate how likely a certain scenario is given the history of data.[19] Evaluation exercises related to forecast densities, such as those in Adolfson *et al*. (2005) and Cogley *et al*. (2005), are accordingly readily conducted within this framework. Given the flexibility of the methodology and its strong statistical foundations, it represents a potential improvement compared to methods currently used by Sveriges Riksbank and the Bank of England to generate fan charts.

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References

Adolfson, M., Lindé, J., Laséen, S. and Villani, M. (2005), "Are Constant Interest Rate Forecasts Modest Interventions?", *International Finance* 8, 509-535.

Amisano, G. and Giannini, C. (1997), *Topics in Structural VAR Econometrics*. Second edition. Springer Verlag, Heidelberg.

Bernanke, B. (1986), "Alternative Explanations of the Money-Income Correlation", *Carnegie-Rochester Conference Series on Public Policy* 25, 49-100.

Blanchard, O. J. (1989), "A Traditional Interpretation of Macroeconomic Fluctuations", *American Economic Review* 79, 1146-1164.

[19] See for example Leeper and Zha (2003).

Blanchard, O. J. and Quah, D. (1989), "The Dynamic Effects of Aggregate Demand and Supply Disturbances", *American Economic Review* 79, 655-673.

Blix, M. and Sellin, P. (1998), "Uncertainty Bands for Inflation Forecasts", Working Paper No. 65, Sveriges Riksbank.

Camarero, M., Ordóñes, J. and Tamarit, C. R. (2002), "Monetary Transmission in Spain: A Structural Cointegrated VAR Approach", *Applied Economics* 34, 2201-2212.

Clements, M. P. (2004), "Evaluating the Bank of England Density Forecasts of Inflation", *Economic Journal* 114, 844-866.

Cogley, T., Morozov, S. and Sargent, T. J. (2005), "Bayesian Fan Charts for U.K. Inflation: Forecasting and Sources of Uncertainty in an Evolving Monetary System", *Journal of Economic Dynamics and Control* 29, 1893-1925.

Creel, J. (2006), "The 'Price Puzzle' Once More", *Applied Economics Letters* 13, 183-187.

Croushore, D. and Stark, T. (2002), "Forecasting with a Real-Time Data Set for Macroeconomists", *Journal of Macroeconomics* 24, 473-484.

Crowder, W. J. and Wohar, M. E. (2004), "A Cointegrated Structural VAR of the Canadian Economy", *Applied Economics* 36, 195-213.

Diebold, F. X., Gunther, A. and Tay, K. (1998), "Evaluating Density Forecasts with Application to Financial Risk Management', *International Economic Review* 39, 863–883.

Diebold, F. X., Tay, K. and Wallis, K. F. (1999), "Evaluating Density Forecasts of Inflation: The Survey of Professional Forecasters". In: Engle, R. F. and White, H. (eds), *Cointegration, Causality and Forecasting: A Festschrift in Honor of Clive W. J. Granger*. Oxford University Press, Oxford.

Doan, T., Litterman, R. and Sims, C. (1984), "Forecasting and Conditional Projection Using Realistic Prior Distributions", *Econometric Reviews* 3, 1-100.

Faust, J. and Henderson, D. (2004), "Is Inflation Targeting Best Practice Monetary Policy?", *Federal Reserve Bank of St. Louis Review* 86, 117-144.

Faust, J and Leeper, E. M. (2005), "Forecasts and Inflation Reports: An Evaluation", Manuscript, Department of Economics, Indiana University.

Froot, K. A. and Rogoff, K. (1995), "Perspectives on PPP and Long-Run Real Exchange Rates". In: Grossman, G. and Rogoff, K. (eds), *Handbook of International Economics*, Vol. 3. North-Holland, Amsterdam.

Gelman, A., Carlin, J. B., Stern, H. S. and Rubin, D. B. (2003), *Bayesian Data Analysis*. Second edition. Chapman and Hall, New York.

Grilli, V. and Roubini, N. (1996), “Liquidity Models in Open Economies: Theory and Empirical Evidence”, *European Economic Review* 40, 847-859.

Groen, J. J. J. (2004), “Corporate Credit, Stock Price Inflation and Economic Fluctuations”, *Applied Economics* 36, 1995-2006.

Hannan, E. J. and Quinn, B. G. (1979), “The Determination of the Order of an Autoregression”, *Journal of the Royal Statistical Society* 41, 190-195.

Jacobson, T., Jansson, P., Vredin, A. and Warne, A. (2001), “Monetary Policy Analysis and Inflation Targeting in a Small Open Economy: A VAR Approach”, *Journal of Applied Econometrics* 16, 487-520.

Kim, S. and Roubini, N. (2000), “Exchange Rate Anomalies in the Industrial Countries: A Solution with a Structural VAR Approach”, *Journal of Monetary Economics* 45, 561-586.

Lawrence, M. J., Edmundson, R. H. and O’Connor, M. J. (1986), “The Accuracy of Combining Judgemental and Statistical Forecasts”, *Management Science* 32, 1521-1532.

Leeper, E. M. (2003), “An Inflation Reports Report”, *Sveriges Riksbank Economic Review* 2003:3, 94-118.

Leeper, E. M. and Zha, T. (2003), “A Theory of Modest Policy Interventions”, *Journal of Monetary Economics* 50, 1673-1700.

Leeper, E., Gordon, D., Sims, C. and Zha, T. (1996), “What Does Monetary Policy Do?”, *Brookings Papers on Economic Activity* 1996:2, 1-78.

Litterman, R. B. (1986), “Forecasting with Bayesian Vector Autoregressions – Five Years of Experience”, *Journal of Business and Economic Statistics* 5, 25-38.

Lucchetti, R. (2006), “Identification of Covariance Structures”, *Econometric Theory* 22, 235-257.

McNees, S. K. (1990), “The Role of Judgment in Macroeconomic Forecasting Accuracy”, *International Journal of Forecasting* 6, 287-299.

Murray, C. and Papell, D. (2005), “The Purchasing Power Parity Puzzle is Worse than You Think”, *Empirical Economics* 30, 783-790.

Orphanides, A. and van Norden, S. (2002), “The Unreliability of Output-Gap Estimates in Real Time”, *Review of Economics and Statistics* 84, 56-583.

Österholm, P. (2004), “Mean Reversion in Swedish Macroeconomic Time Series – Evidence Using a New Panel Data Approach”, *European Review of Economics and Finance* 3, 23-36.

Perron, P. (1989), “The Great Crash, the Oil Price Shock and the Unit Root Hypothesis”, *Econometrica* 57, 1361-1402.

Robertson, J. C. and Tallman, E. W. (2001), "Improving Federal-Funds Rate Forecasts in VAR Models Used for Policy Analysis", *Journal of Business and Economics Statistics* 19, 324-330.

Said, S. E. and Dickey, D. A. (1984), "Testing for Unit Roots in Autoregressive Moving Average Models of Unknown Order", *Biometrika* 71, 599-607.

Shapiro, M. D. and Watson, M. (1988), "Sources of Business Cycle Fluctuations", *NBER Macroeconomics Annual* 3, 111-148.

Sims, C. A. (1980), "Macroeconomics and Reality", *Econometrica* 48, 1-48.

Sims, C. A. (1992), "Interpreting the Macroeconomic Time Series Facts: The Effects of Monetary Policy", *European Economic Review* 36, 975-1000.

Svensson, L. E. O. (2005), "Monetary Policy with Judgement: Forecast Targeting", *International Journal of Central Banking* 1, 1-54.

Svensson, L. E. O. and Williams, N. (2005), "Monetary Policy with Model Uncertainty: Distribution Forecast Targeting", NBER Working Paper No. 11733.

Sveriges Riksbank. (2005), *Inflation Report 2005:1*.

Tierny, L. (1994), "Markov Chains for Exploring Posterior Distributions", *Annals of Statistics* 22, 1701-1762.

Uhlig, H. (2005), "What are the Effects of Monetary Policy on Output? Results from an Agnostic Identification Procecure", *Journal of Monetary Economics* 52, 381-419.

Villani, M. (2005), "Inference in Vector Autoregressive Models with an Informative Prior on the Steady State", Working Paper No. 181, Sveriges Riksbank.

Villani, M. and Warne, A. (2003), "Monetary Policy Analysis in a Small Open Economy Using Bayesian Cointegrated Structural VARs", Working Paper No. 296, European Central Bank.

Waggoner, D. F. and Zha, T. (2003), "Likelihood Preserving Normalization in Multiple Equation Models", *Journal of Econometrics* 114, 329-347.

Appendix A – Unit root tests

Table A1. Results from univariate unit root tests on individual time series.

	Level	First difference
y_t^f	-1.987	-4.893**
$\pi_t^{cpi,f}$	-2.829*	-2.524**
i_t^f	-3.725**	-5.122**
u_t	-1.921	-2.143**
y_t	-2.014	-4.580**
w_t	-1.127	-6.862**
π_t^{cpi}	-2.243	-6.723**
i_t	-0.143	-5.418**
q_t	-1.927	-8.206**

***significant at the 5% level;* * *significant at the 10% level*
The Hannan-Quinn (1979) information criterion has been used to establish lag length in the Augmented Dickey-Fuller test.

Appendix B - Estimates of persistence

Table A2. Persistence measures for the individual time series.

	Equation (A1)		Equation (A2)	
	Persistence	Half-life in quarters	Persistence	Half-life in quarters
Δy_t^f	0.458	1	0.456	1
$\pi_t^{cpi,f}$	0.947	13	0.904	7
i_t^f	0.924	9	0.793	3
u_t	0.983	41	0.970	23
Δy_t	0.449	1	0.448	1
Δw_t	0.338	1	0.178	0
π_t^{cpi}	0.948	13	0.860	5
i_t	0.995	144	0.807	3
q_t	0.945	12	0.854	4

Persistence is measured as the sum of the AR coefficients from univariate regressions of the type

$$x_t = \alpha + \phi_1 x_{t-1} + \ldots + \phi_p x_{t-p} + \varepsilon_t \qquad \text{(A1)}$$

and

$$x_t = \alpha + \phi_1 x_{t-1} + \ldots + \phi_p x_{t-p} + \theta D_t + \varepsilon_t . \qquad \text{(A2)}$$

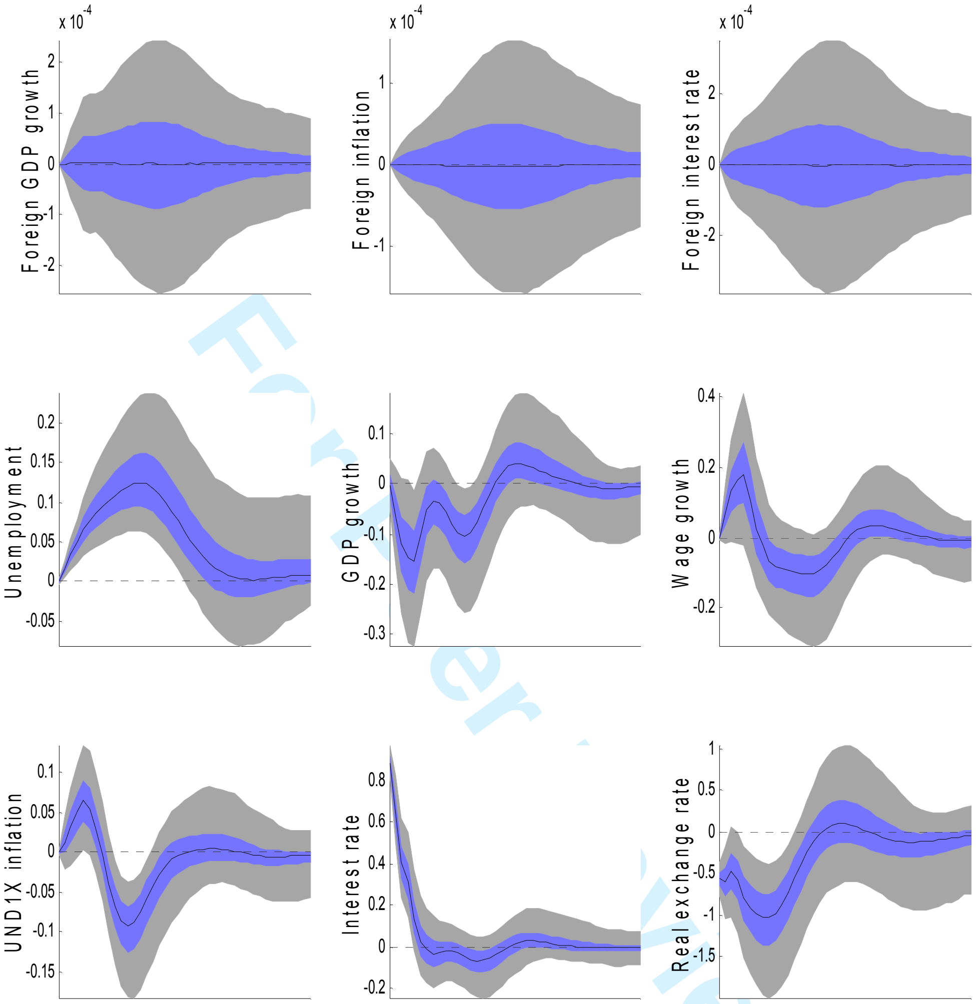
x_t is the transformation of the variable appearing in Table A2 and D_t is a dummy variable which takes on the value 1 between 1980Q1 and 1992Q4 and 0 otherwise. Lag length in the regressions is based on the Augmented Dickey-Fuller test and accordingly set to *p* when *p*-1 lagged differences were selected for the Augmented Dickey-Fuller test.

Following Murray and Papell (2005), the half-life of a shock to a series is then calculated as

$$H = \frac{\ln(0.5)}{\ln\left(\sum_{i=1}^{p} \phi_i\right)} . \qquad \text{(A3)}$$

Appendix C – Impulse response functions

Figure A1. Impulse response functions of variables in the system with respect to Swedish monetary policy shock.



Black line is the median impulse. Coloured bands are 50 and 90 percent confidence bands.